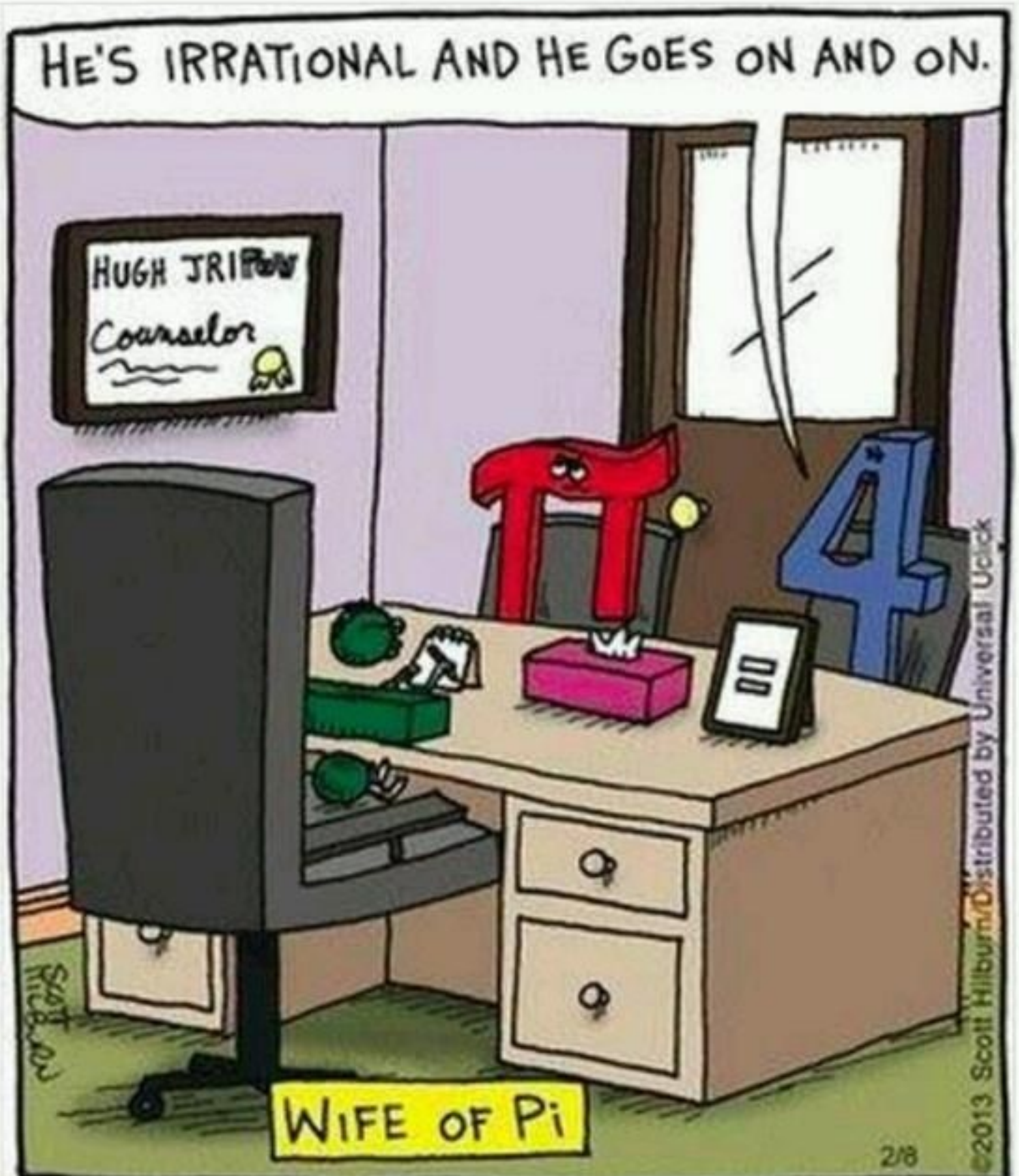
Unit 2: Roots and Powers

Math 10: Foundations & Pre Calculus
Textbook: Chapter 4 (pg 204-249)

must have a calculator


HE'S IRRATIONAL AND HE GOES ON AND ON.
HUGH JRI
Counselor
WIFE OF Pi
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Part 1: Introduction to Radicals

Checklist (Objectives)

- Difference between rational and irrational numbers
- Determine if a number or radical is rational or irrational.
- Order irrational numbers
- Introduction to radicals & types of roots

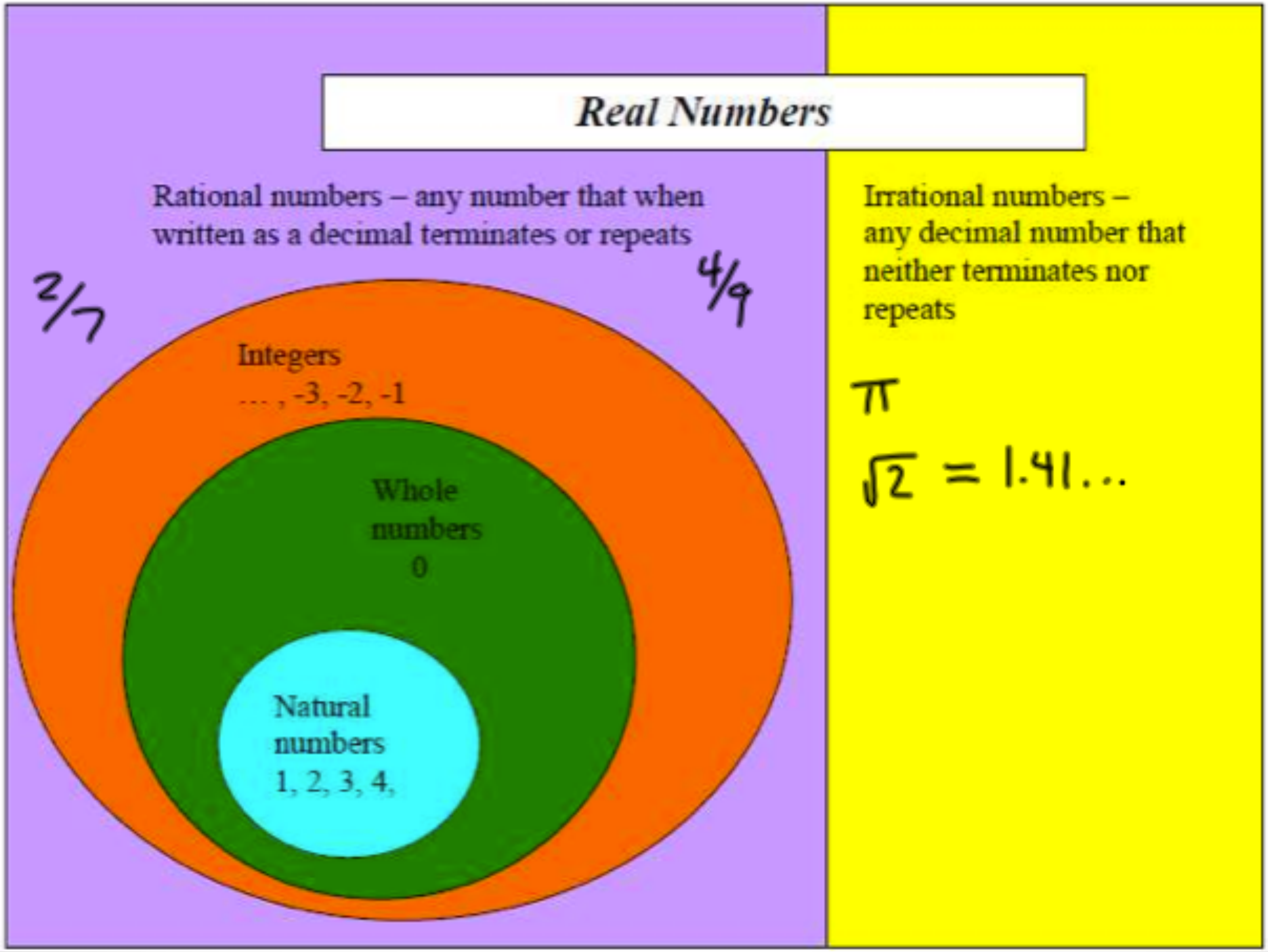
**If you are having any troubles with the new vocabulary, make sure to look at your "checklist".

**Your checklist is for self evaluation.

The Number System

Natural Numbers (N): counting numbers
Whole Numbers (W): the natural numbers and zero
Integers (I): the positive and negative whole numbers
Rational Numbers: any number that can be written as $\frac{m}{n}$, where m and n are integers (a fraction or a decimal that terminates or repeats).
Irrational Numbers: any number that cannot be written as a fraction (a non terminating or non repeating decimal)
Real Numbers: The sets of all irrational and rational numbers

Another way to look at it…



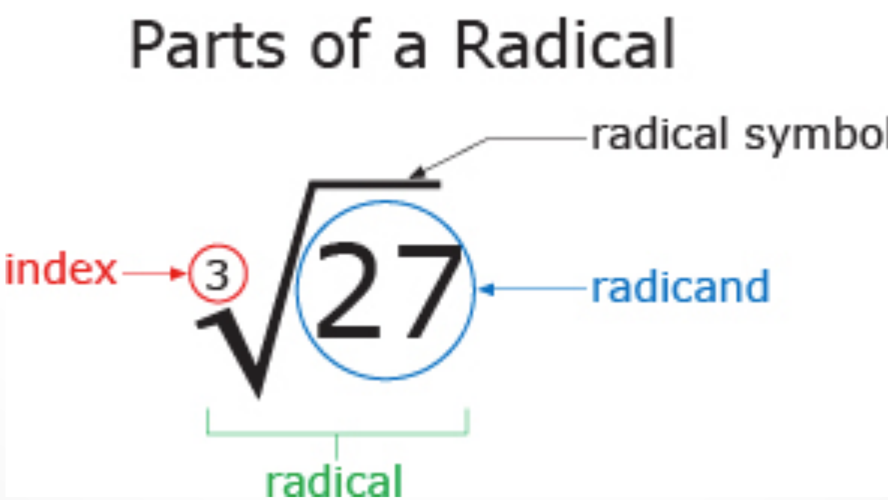
Rational Numbers	Irrational Numbers
-if the number is written as a quotient of integers (fraction)	-number cannot be written as a fraction
-If when taking the root of the number it is a perfect square, cube, fourth, etc.	-decimal goes on without repeating
-if the decimal terminates or repeats	Ex: **π =3.14159265358979323846264338 32795 (and more…)**

History of Real Numbers

- The ancient greek mathematician *Pythagoras* believed that all numbers were **rational** (could be written as a fraction).
- One of his students *Hippasus* proved (using geometry, it is thought) that you could **not** represent the square root of 2 as a fraction, and so it was *irrational*.
- The discovery of irrational numbers is said to have been shocking to the Pythagoreans, and Hippasus is supposed to have drowned at sea, apparently as a punishment from the gods.

Introduction to Radicals

- A **radical** is the expression of the form $\sqrt[n]{x}$ where n is a natural number.
- The **radical** can be any root, whether a square root, a cube root, or some other root.
- n is called the index and indicates which root is being taken. If the index is not written it is assumed to be 2.



Exact or Approximate Value?

- Radicals that are square roots of perfect squares, cube roots of perfect squares, and so on are rational numbers and are the exact value

- When an irrational number is written as a radical, the radical is the exact value of the irrational number. We can use the root keys on our calculator to find the approximate value.

- Examples:

$\sqrt{2}$	$\sqrt[3]{-50}$	$\sqrt[3]{27}$
1.414213562..	-3.684031499...	3

Example 1: Label the following as rational or irrational. Is the root form or the decimal form the exact value of the numbers?

a) $\sqrt{3}$

b) $\sqrt{16}$

c) $\sqrt{20}$

d) π

e) $\sqrt{\frac{4}{9}}$

f) $\sqrt{\frac{4}{5}}$

Ordering Irrational Numbers

Example 1: Use a number line to order these numbers from least to greatest.

$\sqrt[3]{50}$ $\sqrt[3]{8}$ π 1.3, $\sqrt[3]{27}$

Way 1: Estimation

Way 2: Convert to Decimal Form (using calculator-show your work).

Assignment 2.1 : Pg 211/212

#3, 5, 6, 7b, 10 a) use estimation b) use calculator, 12

Part 2: Mixed and Entire Radicals

<u>Checklist (Objectives)</u>

- Express entire radicals as mixed radicals
- Understanding of the multiplication property of radicals
- I am able to turn a mixed radical back into an entire radical

Make Connections

We can name the fraction $\frac{3}{12}$ in many different ways:

$\frac{1}{4}$ $\frac{5}{20}$ $\frac{30}{120}$ $\frac{100}{400}$

How do you show that each fraction is equivalent to $\frac{3}{12}$?

Why is $\frac{1}{4}$ the simplest form of $\frac{3}{12}$?

Just as with fractions, equivalent expressions for any number have the same value.

- $\sqrt{16 \cdot 9}$ is equivalent to $\sqrt{16} \cdot \sqrt{9}$ because:

Multiplication Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b},$$

where n is a natural number, and a and b are real numbers

We can use this property to simplify square root and cube roots that are not perfect squares or perfect cubes, but have factors that are perfect squares or perfect cubes.

Example 1 : Simplify $\sqrt{24}$

Step 1: Does the number have a factor that is a perfect square?

We can simplify $\sqrt{24}$ because 24 has a perfect square factor of 4.

Step 2: Rewrite 24 as the product of two factors, one of which is your perfect square.

$$\sqrt{24} = \sqrt{4 \cdot 6} = \sqrt{4} \cdot \sqrt{6} = 2 \cdot \sqrt{6} = 2\sqrt{6}$$

We read $2\sqrt{6}$ as "2 root 6"

Example 2: Simplify $\sqrt[3]{24}$

Example 3: Simplify $\sqrt{80}$

Can we simplify $\sqrt[4]{24}$?

Writing Mixed Radicals as Entire Radicals

Step 1: Take the whole number and put it to the exponent that is the same as the type of root.

Step 2: Write this number under a root sign

Step 3: Multiply the root numbers together to get an entire radical.

Example 1:

$$7\sqrt{3}$$

$$\sqrt{7^2}\cdot\sqrt{3}$$

$$\sqrt{49}\cdot\sqrt{3}$$

$$\sqrt{147}$$

Write each mixed radical as an entire radical.

Example 2: $2\sqrt[3]{4}$

Example 3: $2\sqrt[5]{3}$

Assignment 2.2: Pg 218/219

#4 odd, #5 odd, 9, 11, 12 (f, h, I, j), 21